⑰ $y = x^{1/3}(x-4) = x^{4/3} - 4x^{1/3}$

$(-2)^{4/3} - 4(2)^{1/3}$

$y' = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3}$

$y'' = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = 0$

$\frac{4}{9\sqrt[3]{x^2}} + \frac{8}{9\sqrt[3]{x^5}} = 0$

$\frac{4}{9\sqrt[3]{x^2}} = -\frac{8}{9\sqrt[3]{x^5}}$

$36\sqrt[3]{x^5} = -72\sqrt[3]{x^2}$

$\sqrt[3]{x^5} = -2\sqrt[3]{x^2}$

$\left(\frac{\sqrt[3]{x^5}}{\sqrt[3]{x^2}}\right)^3 = (-2)^3$

$x^3 = -8$

$x = -2, 0$

$\frac{x^5}{x^2} = -8$

$x^5 = -8x^2$

$x^5 + 8x^2 = 0$

$x^2(x^3+8) = 0$

$\frac{4}{9\sqrt[3]{9}} + \frac{8}{9\sqrt[3]{243}}$

.21 -.14

+ − +

-2 0

(-2,)

(0, 0)

Nov 9-10:10 AM

Opener

Exploration #2 on pg 214

EXPLORATION 2 **Finding f from f' and f''**

A function f is continuous on its domain $[-2, 4]$, $f(-2) = 5$, $f(4) = 1$, and f' and f'' have the following properties.

x	$-2 < x < 0$	$x = 0$ (max)	$0 < x < 2$	$x = 2$	$2 < x < 4$
f'	inc + $\overset{+}{\text{slope}}$	does not exist	dec − $\overset{-}{\text{slope}}$	0	dec − $\overset{-}{\text{slope}}$
f''	conc up +	does not exist	conc up +	0 inf point	conc down −

1. Find where all absolute extrema of f occur.
   $x = 4$ min $x = 0$ max
2. Find where the points of inflection of f occur.
   $x = 2$
3. Sketch a possible graph of f.

Oct 24-12:30 PM

4-3 day 3 The Second Derivative Test for Local Extrema

Learning Objectives:

I can use the second derivative test to find local extrema of a function.

Oct 24-11:45 AM

Ex1. Given the function

$$f(x) = 2x^3 + 3x^2 - 12x + 6$$

a.) Find the critical points.

Candidates: 1, -2, -½

$f'(x) = 6x^2 + 6x - 12$

$6(x^2 + x - 2) = 0$

$6(x-1)(x+2) = 0$

$x = 1, -2$

+ − +

-2 1

max min

$f''(x) = 12x + 6 = 0$

$x = -\frac{1}{2}$

− +

$-\frac{1}{2}$ inf. pt.

b.) Find the second derivative at each of these points.

∪ $x = 1$ 18 $12x + 6 = 0$

∩ $x = -2$ -18

Oct 24-11:45 AM

x	f	f''	Graph
x=1			
x=2			

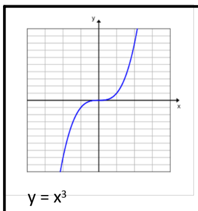
Oct 24-11:45 AM

The 2nd Derivative Test for Classifying Local Extrema

- If f′(c) = 0 and f″ (c) > 0, then x = c is a local minimum.
- If f′(c) = 0 and f″ (c) < 0, then x = c is a local maximum.

However, if f′(c) = 0 and f″ (c) = 0, then the 2nd der. test is inconclusive

Oct 24-11:45 AM

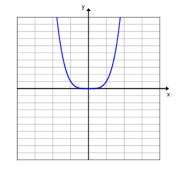


$y = x^3$

$y' = 3x^2$ @x = 0, $y' = 0$

$y'' = 6x$ @x = 0, $y'' = 0$

Inflection Point



$y = x^4$

$y' = 4x^3$ @x = 0, $y' = 0$

$y'' = 12x^2$ @x = 0, $y'' = 0$

Minimum

If f′(c) = 0 and f′′(c) = 0, the Second Derivative Test is inconclusive.

Oct 24-11:45 AM

Ex1. Find and classify the extrema $f(x) = 1 + x - x^2 - x^4$

Find extrema, inflection points, intervals where function is decreasing/increasing and intervals where graph is concave up/concave down.

$f' = 1 - 2x - 4x^3$

$f'' = -2 - 12x^2 = 0$

$x^2 = \frac{-1}{6}$

no inf. pts.

inc $(-\infty, .385)$

dec $(.385, \infty)$

$x = .385$

$+ \mid -$

.385

concave down everywhere

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Homework

pg 215 # 25-30, 33, 36-40, 48, 51, 52, 55-59

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